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The vDVZ Discontinuity, DGP Gravity, and Cosmology

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Abstract We discuss some properties of the van Dam–Veltman–Zakharov (vDVZ) discontinuity in the Dvali–Gabadadze–Porrati (DGP also called brane-induced) gravity. This model exhibits a vDVZ discontinuity when linearized over a Minkowski background. However, one can show that this discontinuity disappears on general Friedmann–Lemaître–Robertson–Walker space-times, when the radius of transition between 4D and 5D gravity is sent to infinity with respect to the background Hubble radius. This radius of transition plays for the brane-induced gravity model a rôle equivalent to the Compton wavelength of the graviton in a Pauli–Fierz theory, as far as the vDVZ discontinuity is concerned. These results were obtained considering the Cauchy problem associated with linearized DGP model over a cosmological background and provide a novel way to address the issue of the vDVZ discontinuity, as well as to look at some properties of the DGP model. Some parts of the discussion have not been presented before and comments are also made on the issue of the growth of cosmological perturbations in the DGP model.

1 Introduction

Brane world models offer several interesting new ways to modify the gravitational interaction and mimic a 4D gravity theory from an intrinsically higher dimensional one. This can be achieved assuming that the $(4+n)$ dimensional bulk space-time has a special topology, with e.g. compact extra-dimensions [1–3], or geometry, being e.g. a patch of AdS_5 space-time [4], and leads generically to modifications of the gravitational interaction at short distances. Variations of these models have been considered where gravity was modified at large (cosmological) distances, being mediated by a resonance of massive graviton propagating

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over an *infinite volume* bulk [5, 6]. The Dvali–Gabadadze–Porrati model [7] shares similar properties. This model has been applied to cosmology with the aim of explaining the observed late time acceleration of the universe by a large distance modification of gravity [8–10]. Aside its interesting phenomenology, the Dvali–Gabadadze–Porrati (DGP in the following) model is also interesting as a consistent framework to study various long standing problems associated with *massive gravity*. Here we are mostly reporting on a work [11] studying the van Dam–Veltman–Zakharov (vDVZ) discontinuity [12–14] over various backgrounds. Those will be cosmological, but not necessarily maximally symmetric, Friedmann–Lemaître–Robertson–Walker (FLRW) space-times.

The essence of the vDVZ discontinuity is that the quadratic Pauli–Fierz [15, 16] action for a massive graviton on a Minkowski background gives different physical predictions (like e.g. for light bending) from linearized general relativity whatever the smallness of the graviton mass. It is known, however, that this is no longer true on a maximally symmetric nonflat background [17–19], when the graviton Compton wavelength (squared) is much larger than the background cosmological constant. The same is expected to be true for more general space-times with nonvanishing curvature, where some curvature scale is supposed to play the role of the cosmological constant (square-rooted). Here we will use results [20, 21] for brane-world cosmological perturbations to discuss this issue in the DGP model, in which the graviton propagator has the same tensorial structure as in Pauli–Fierz theory. Our discussion will be presented in a way not usually adopted for addressing the vDVZ discontinuity: the Cauchy problem. Our presentation will be based on the original reference [11], but some part of it will be new.

In [11], it was shown that the linearized DGP theory over an arbitrary FLRW space-time has a well defined limit when the radius of transition between 4D and 5D gravity is sent to infinity with respect to the background Hubble radius. This radius of transition plays for the DGP model a rôle equivalent to the Compton wavelength of the graviton in a Pauli–Fierz theory, as far as the vDVZ discontinuity is concerned. This well defined limit was shown to obey the linearized 4D Einstein's equations whenever the Hubble factor is nonvanishing. This shows the disappearance of the vDVZ discontinuity for general FLRW background, and extends the above mentioned result for maximally-symmetric space-times. Our reasoning is valid for matter with simple equation of state such as a scalar field, or a perfect fluid with adiabatic perturbations, and involves to distinguish between brane space-times with a nonvanishing scalar curvature and brane space-times with a vanishing one. Here, only the first case will be discussed. We refer the reader to reference [11] for the discussion of the second case, as well as for issues related to the breakdown of linear perturbation theory.

The paper is organized as follows. In the remaining of this introduction we remind aspects of the vDVZ discontinuity (Sect. 1.1), introduce the DGP model (Sect. 1.2) as well as the solutions for the cosmological background that will be used later (Sect. 1.3). In a second section, we discuss the Cauchy problem for a scalar toy model of DGP gravity, in a way suitable for comparison with the following discussion. We then turn to discuss the cosmological perturbations of the DGP model (Sect. 3). First, we introduce the general formalism that we will later use (Sect. 3.1), then we apply it to the study of the vDVZ discontinuity over Minkowski background (Sect. 3.2). Next we turn to the case of time dependent perturbations over more general backgrounds, first by recalling the differential problem associated with this case (Sect. 3.3), then, by discussing FLRW backgrounds with nonvanishing Ricci scalars (Sect. 3.4). Last, we summarize and conclude (Sect. 4).

1.1 vDVZ Discontinuity and Pauli–Fierz Theory

The van Dam–Veltman–Zakharov discontinuity [12–14] was first discussed in the framework of the Pauli–Fierz theory for a massive spin two field [15, 16]. This theory can be defined by the following action

$$S^{(\Lambda_{(4)},m)} \equiv S^{(2)}_{\rm EH} + S^{(m)}_{\rm PF} + S_{\rm MC}, \tag{1}$$

where $S^{(2)}_{\rm EH}$ is the Einstein–Hilbert action truncated to quadratic order into a small metric fluctuation $h_{\mu\nu}$, considered as a dynamical field over some background metric $\bar{g}_{\mu\nu}$ solution to Einstein's equations, $S^{(m)}_{\rm PF}$ is the Pauli–Fierz mass term, and $S_{\rm MC}$ is the action defining the coupling between the graviton and matter. The Pauli–Fierz mass term is defined by

$$S^{(m)}_{\rm PF} \equiv \frac{m^2}{8\kappa^2_{(4)}} \int d^4x \sqrt{-\bar{g}} h_{\mu\nu} h_{\alpha\beta} (\bar{g}^{\mu\nu} \bar{g}^{\alpha\beta} - \bar{g}^{\alpha\mu} \bar{g}^{\beta\nu}), \tag{2}$$

where m is a mass parameter and $\kappa^2_{(4)}$ is the inverse squared reduced 4D Planck mass. On a flat background, this mass term gives a mass to the spin two field $h_{\mu\nu}$ in a ghost-free way. The coupling term is then defined as the standard coupling between $h_{\mu\nu}$ and a (conserved) energy momentum tensor $T_{\mu\nu}$. Note that by definition the action $S^{(\Lambda_{(4)},0)}$ is just the quadratic action for a massless graviton coupled to matter and deduced from general relativity. Wishing to compare the massless ($S^{(\Lambda_{(4)},0)}$) and the massive ($S^{(\Lambda_{(4)},m\neq 0)}$) theory, it is convenient to define the amplitude $\mathcal{A}^{(\Lambda_{(4)},m)}$ due to one particle exchange between two conserved energy momentum tensors $T_{\mu\nu}$ and $S_{\mu\nu}$. This amplitude is given formally by

$$\mathcal{A}^{(\Lambda_{(4)},m)} \equiv \frac{1}{2} \int d^4x\, S^{\mu\nu}(x) h_{\mu\nu}(T)(x). \tag{3}$$

Let us first consider the case where the background metric $\bar{g}_{\mu\nu}$ is the Minkowski metric $\eta_{\mu\nu}$ and the cosmological constant $\Lambda_{(4)}$ vanishes. The dramatic observation made in Refs. [12–14] is that, in this case, the two theories $S^{(0,0)}$ and $S^{(0,m\neq 0)}$ give different physics whatever the smallness of the mass parameter m. Considering e.g. nonrelativistic sources separated by a distances sufficiently small with respect to the graviton Compton wavelength, the amplitude due to the exchange of a massive graviton is given approximately by taking the $m \to 0$ limit in $\mathcal{A}^{(0,m\neq 0)}$, that we note $\mathcal{A}^{(0,m\to 0)}$. One finds

$$\mathcal{A}^{(0,m\to 0)} = \frac{4}{3} \mathcal{A}^{(0,m=0)}, \tag{4}$$

so that the massive amplitude stays different from the massless one, whatever the smallness of the graviton mass. This translates into a similar discrepancy in the nonrelativistic potential between the two same sources, the potential of the massive theory being larger by a factor 4/3. This extra attraction can be attributed to the exchange of the helicity zero polarization of the massive graviton (which has 3 more polarizations than the massless one). This difference can be nullified by redefining the Newton constant of the massive theory with respect to the massless one, assuming, e.g., that one measures the Newton constant by some Cavendish experiment. However, with such a rescaling, the discontinuity will then reappear in other observables, like the light bending. The latter will then be 25% smaller in the massive case than in the massless one, which is much too large to be compatible with current measurements of the light bending by the sun [12–14].

The situation is however very different when the background space-time is nonflat. Indeed taking $\bar{g}$ to parameterize a de Sitter or anti de Sitter space-time with a cosmological constant $\Lambda_{(4)}$, one finds that the previously defined amplitude $\mathcal{A}$ is such that

$$\mathcal{A}^{(\Lambda_{(4)}\neq 0, m\to 0)} = \mathcal{A}^{(\Lambda_{(4)}\neq 0, m=0)}, \tag{5}$$

so that the discontinuity disappears on a maximally symmetric background with a nonvanishing cosmological constant [17–19, 22].[1] This shows that the limits $m \to 0$ and $\Lambda_{(4)} \to 0$ do not commute; however the amplitude $\mathcal{A}^{(\Lambda_{(4)},m)}$ goes smoothly toward $\mathcal{A}^{(\Lambda_{(4)},0)}$ when one lets $m^2\Lambda_{(4)}^{-1}$ go to zero.

As was alluded to here-above, the presence of the discontinuity can be attributed to the exchange of the scalar polarization (helicity 0) of the massive graviton. The latter is coupled to the trace of the energy momentum tensor of a conserved source. When the background is Minkowski, this coupling is mass independent and remains in the limit where the graviton mass is sent to zero. However, on a maximally symmetric space-time with nonvanishing curvature, the helicity zero coupling becomes proportional to the graviton mass and disappears in the zero mass limit, allowing to recover the massless result. A similar disappearance can be expected to happen in more general cases where the background space-time has nonvanishing curvature [23, 24]. This is in particular supported by the study of Schwarzschild-like solutions where it has been argued some time ago by Vainshtein [25] that the vDVZ discontinuity could disappear nonperturbatively, i.e. in the full exact solution of the theory. Indeed, as noted by Vainshtein, the Schwarzschild radius enters in combination with the graviton mass at the second nontrivial order in the perturbation theory to open the possibility for a non perturbative recovery of the short distance behavior the standard Schwarzschild solution. In the case of a maximally symmetric background with nonvanishing curvature, however, there is already a length scale (besides the graviton Compton length) at the first order of perturbation theory, the radius of curvature of the background space-time, and, as we saw above, this combines with the graviton mass to lead to a disappearance of the vDVZ discontinuity already at the linear level.

One thus would wish to study the issue of the disappearance of the vDVZ discontinuity for general curved backgrounds. This is however plagued by possible inconsistencies. Indeed, the background metric $\bar{g}_{\mu\nu}$, discussed so far, was introduced as a completely extraneous field with respect to the "graviton" $h_{\mu\nu}$, and one would need to have a fully nonlinear theory of gravity which linearized action would be given by (1). However there is no known nonpathological such a theory. E.g. it has been shown [26] that starting from the full Einstein–Hilbert action, and going beyond the quadratic level in the expansion for the kinetic term, causes a ghost-like sixth degrees of freedom to propagate. Other nonlinear completion using two metrics (The so called *strong gravity* or *bi-gravity* theories [27, 28]) have also been shown to suffer from the same drawbacks [28]. One could also think to start from a higher dimensional theory and truncate it in some clever ways to retain a finite number of massive graviton Kaluza–Klein states, such truncations are however also pathological [29].[2]

Here we would like to concentrate on a nonlinear theory which exhibit a vDVZ discontinuity on flat backgrounds. This is the brane world model of Dvali–Gabadadze–Porrati (DGP) [7] that we will introduce in more details in the next subsection. So far, it has not been

[1] However the theory is nonunitary for $\Lambda_{(4)} > 0$ and $m^2 < 2\Lambda_{(4)}/3$ [18, 19].

[2] Note also that the nonperturbative disappearance à la Vainshtein of the vDVZ discontinuity has also been analyzed in some of the above mentioned nonlinear completion of action (1) with a negative result [30, 31].

found to suffer from the same problems as the other nonlinear completions we mentioned, and in addition, it has known exact cosmological solutions [8]. The aim of this paper is to use those solutions as a consistent background to study the issue of the disappearance of the vDVZ discontinuity over cosmological space-times which are not maximally symmetric.

1.2 DGP Model

The DGP model [7] we are considering is a 5D brane-world model with bulk gravitational action

$$S_{(5)} = -\frac{1}{2\kappa_{(5)}^2}\int d^5X\sqrt{g_{(5)}}R_{(5)}, \tag{6}$$

where $R_{(5)}$ is the 5D Ricci scalar, and $\kappa_{(5)}^2$ is the inverse third power of the reduced 5D Planck mass. To account for the brane, one adds to this action a term taking care of brane-localized fields given by

$$S_{(4)} = \int d^4x\sqrt{g_{(4)}}\mathcal{L}, \tag{7}$$

where $\mathcal{L}$ is a Lagrangian density given by

$$\mathcal{L} = \mathcal{L}_{(M)} - \frac{1}{2\kappa_{(4)}^2}R^{(4)}.$$

In the above expression, $\mathcal{L}_{(M)}$ is a Lagrangian for brane localized matter and $R^{(4)}$ is the Ricci scalar of the induced metric $g_{\mu\nu}^{(4)}$ on the brane defined by

$$g_{\mu\nu}^{(4)} = \partial_\mu X^A\partial_\nu X^B g_{AB}^{(5)}, \tag{8}$$

$X^A(x^\mu)$ are defining the brane position, where X^A are bulk coordinates, and x^μ coordinates along the brane world-volume.[3] We also implicitly include in the action a suitable Gibbons–Hawking term [32] for the brane.

Before discussing the DGP model defined by the sum of actions (6) and (7), let us first turn to a scalar toy model, proposed in the original reference [7], which captures some key features of the full DGP model. It can be defined by the action for the suitably normalized scalar field ϕ,

$$S_\phi = \int d^4x dy\left\{\frac{1}{\kappa_{(5)}^2}\partial_A\phi\partial^A\phi + \frac{1}{2}\delta(y)J_{(4)}\phi + \frac{1}{\kappa_{(4)}^2}\delta(y)\partial_\mu\phi\partial^\mu\phi\right\}. \tag{9}$$

This model is simply one for a scalar field in a 5D flat bulk with a brane localized (in $y=0$) kinetic term added for it, and a brane localized source term $J_{(4)}$. The equations of motion derived from this action read

$$\left(\frac{1}{\kappa_{(5)}^2}\partial^A\partial_A + \frac{1}{\kappa_{(4)}^2}\delta(y)\partial_\mu\partial^\mu\right)\phi = \frac{1}{2}\delta(y)J_{(4)}. \tag{10}$$

[3] Throughout this article, we will adopt the following convention for indices: upper case Latin letters $A, B, \dots$ will denote 5D indices: $0, 1, 2, 3, 5$; Greek letters $\mu, \nu, \dots$ will denote 4D indices parallel to the brane: $0, 1, 2, 3$; lower case Latin letters from the middle of the alphabet: $i, j, \dots,$ will denote space-like 3D indices parallel to the brane: $1, 2, 3$.

In this model, the interaction potential between two static sources, separated by a distance r, interpolates between a 4D $1/r$ behavior at small distances and a 5D $1/r^2$ behavior at large distances, as shown in reference [7]. The crossover distance r_c between the two regimes being is given by

$$r_c = \frac{\kappa_{(5)}^2}{2\kappa_{(4)}^2}. \tag{11}$$

The same behavior was found in Ref. [7] to hold for the gravitational potential between two sources put on the brane in the theory defined by the sum of actions (6) and (7), whenever the background bulk and brane space-times were both taken to be Minkowski space-times. However it was also found there that, from a 4D point of view, gravity is mediated by a continuum of massive Kaluza–Klein modes, with no normalizable zero mode entering into the spectrum. This being a consequence of the bulk being flat and infinite. As a consequence the tensorial structure of the graviton propagator was shown to be the one of a massive graviton and the model exhibits a vDVZ discontinuity.

As discussed in the previous subsection, the vDVZ discontinuity would seem to rule out phenomenological applications of DGP model, if one trusts the linear approximation to compute observables such as the light bending from the sun. However, based on the fact that the exact cosmological solutions (found in [8] and recalled in the next subsection) do not exhibit any sign of the vDVZ discontinuity, it has been suggested that the vDVZ discontinuity of the DGP model was indeed an artifact of the perturbation theory [23] following the similar suggestion made by Vainshtein in the framework of Pauli–Fierz theory [25], and recalled in the previous subsection. This has been studied in different situations by expansions going beyond the linear order in the brane bending [30, 33–36]. Those analysis, mostly concentrated on the case of spherically symmetric solutions on the brane, confirmed so far the original suggestion made in Ref. [23]: for a spherically symmetric metric on the brane, the solution given by the linearized theory breaks down below a nonperturbative distance, given by[4]

$$r_v = (r_c^2 r_S)^{1/3}. \tag{12}$$

This scale can be coined from the equivalent one found by Vainshtein to appear for a Pauli–Fierz model [25]. It depends on a curvature scale of the spherically symmetric metric, namely the 4D Schwarzschild radius r_S. For distances much lower than r_v it has been found that the spherically symmetric solution on the brane is close to a usual 4D Schwarzschild metric (with 4D, massless, "tensorial structure") so that there is no more discontinuity. In particular, one sees that r_v diverges as r_c goes to infinity (this is similar to sending m to zero in Pauli Fierz action), the 4D parameters ($\kappa_{(4)}^2$, and the mass of the source) being fixed. This disappearance of the vDVZ discontinuity can be attributed to the fact that one mode entering into the gravitational exchange, and related to the brane bending (or the brane extrinsic

[4] Note however that all the spherically symmetric solutions studied so far are approximate solutions. This has several aspects. First, the work [36] suggests that the linearized theory is never a good approximation of the exact nonperturbative solution of the brane (even if this work also finds the appearance of the scale (12) as well as a nonperturbative recovery of the massless tensorial structure below that scale). Second, the fact that the exact spherically symmetric solution is not known still allows a loophole to all analysis done so far. One could indeed argue that pathologies are arising in the exact solution, similar to those found in Ref. [31] in the context of bi-gravity. However the situation of the DGP model with respect to this issue is as good (or as bad) as in any other brane world models (like the Randall Sundrum model [4]) where the exact (stable) Schwarzschild-type solution on the brane, corresponding to the perturbative analysis, is not known.

curvature), has a cubic interaction which becomes important at distances lower than r_v, as discussed in detail in [23]. This will not matter for us here, since we will only be concerned with the linear perturbation theory, and, following the discussion of the previous subsection as well as what is suggested by the study of spherically symmetric solutions, ask whether already at the linear level, the vDVZ discontinuity disappears in the DGP model on cosmological background, when we let r_c go to infinity and keep the parameters of the $4D$ part of the action, (7), fixed.

We thus need to look at the DGP equations of motion linearized over a cosmological background. That is to say we need to look at the so-called *cosmological perturbations* of the DGP model. We will only consider the case of scalar perturbations (as seen from an observer on the brane) since it is for those perturbations that the discontinuity is expected to show up, being related to the helicity-zero mode of the graviton. In the next subsection, we discuss some properties of the cosmological background we are interested in.

1.3 Cosmological Background for DGP Model

Simple cosmological solutions of the DGP model are known exactly. In a so-called Gaussian Normal coordinate system the line element of the bulk metric takes the form

$$ds_{(5)}^2 = -n^2(t, y)dt^2 + a^2(t, y)\delta_{ij}dx^i dx^j + dy^2, \tag{13}$$

while the metric on the brane is parameterizing a Friedmann–Lemaître–Robertson–Walker (FLRW) space-time with a scale factor $a_{(b)}$. This scale factor is a solution of the following modified Friedmann equation (considering here only the case of flat spatial sections)

$$\dot{\rho}_{(M)} = -3H(P_{(M)} + \rho_{(M)}), \tag{14}$$

$$1 = -\frac{\Upsilon}{2} + \sqrt{\frac{\kappa_{(4)}^2 \rho_{(M)}}{3H^2} + \frac{\Upsilon^2}{4}}, \tag{15}$$

where we have defined the parameter Υ by

$$\Upsilon = \frac{\eta}{Hr_c}, \tag{16}$$

where $H \equiv \dot{a}_{(b)}/a_{(b)}$ is the brane Hubble parameter, and η is the sign[5] of the brane effective energy density $\rho \equiv \rho_{(M)} - 3H^2/\kappa_{(4)}^2$, and $\rho_{(M)}$ and $P_{(M)}$ are respectively the brane localized matter energy density and pressure. The early time behavior of the cosmology defined by (14) and (15) is obtained by considering the limit $\Upsilon \to 0$. In this limit (which means that the Hubble radius H^{-1} is much smaller than the crossover radius r_c), (15) is identical to the usual first Friedmann equation and one recovers standard cosmology (with no sign of the vDVZ discontinuity, as recalled above). At late time, however, one has deviations from standard cosmology.

2 The Cauchy Problem in Scalar Toy Model for DGP Gravity

In the next section we will discuss the vDVZ discontinuity from the point of view of the Cauchy problem for scalar cosmological perturbations. Here we first discuss the similar

[5] η, which takes the values ± 1, is equal to $-\epsilon$ of Ref. [8].

problem for the toy model of action (9) that we will further simplify reducing it to two dimensions. This leads to the following equations of motion

$$\ddot{\phi} - \phi'' = 0, \tag{17}$$

$$\phi' - r_c\ddot{\phi} = 0 \quad \text{at } y = 0, \tag{18}$$

where a dot and a prime mean respectively a derivative with respect to time t and y coordinates. The "brane" is thus the $y = 0$ line. We consider solving this system in the first quadrant $y \geq 0, t \geq 0$ subject to chosen initial data on the $t = 0$, $y \geq 0$ half line. Those initial data are specified by

$$\phi(y \geq 0, t = 0) = f(y), \tag{19}$$

$$\dot{\phi}(y \geq 0, t = 0) = g(y), \tag{20}$$

where f and g are some given functions that can be specified at will (provided they fulfill in $y = 0, t = 0$ some constraints to be compatible with (17) and (18) at the origin). It is then easily seen that the solution of this problem in the upper triangle $y \geq 0, t \geq 0, t - y \geq 0$ is given by

$$\phi(t, y) = \frac{1}{2}f(t + y) + \frac{1}{2}f(0) + \frac{1}{2}\int_{z=0}^{z=t+y} g(z)dz \\ + r_c\frac{\dot{f}(0) - g(0)}{2}(e^{-(t-y)/r_c} - 1) + G_P(r_c, t - y) \tag{21}$$

where $G_P(r_c, z)$ is the solution of the ordinary differential equation

$$\frac{d^2G_P(r_c, z)}{dz^2} = -\frac{1}{r_c}\frac{G_P(r_c, z)}{dz} - \frac{1}{2}\frac{dh(z)}{dz} + \frac{1}{2r_c}h(z), \tag{22}$$

where $h(z)$ is defined by $h(z) = \frac{df(z)}{dz} + g(z)$, and $G_P(r_c, z)$ obeys the initial condition $G_P(r_c, 0) = 0$ and $\frac{dG_P(r_c,z)}{dz}_{|z=0} = 0$. Let us then consider the $r_c \to \infty$ limit of the expression (21) for ϕ on the brane at $y = 0$. This limit is given by

$$\phi(t, y = 0)_{|r_c->\infty} = f(0) + g(0)t, \tag{23}$$

that is to say the solution of the ordinary differential equation $\ddot{\Phi} = 0$ with initial conditions

$$\Phi(0) = f(0), \tag{24}$$

$$\dot{\Phi}(0) = g(0). \tag{25}$$

Those initial conditions are the restriction on the brane of the initial conditions (19) and (20). This agree with the expectation: in the $r_c \to \infty$ limit the solution of the Cauchy problem associated with the toy model (9) is such that it is given on the brane by the solution of the Cauchy problem derived from the original one, by restricting on the brane both the initial conditions and the equation of motion. The latter equation of motion is thus the one of a one dimensional field. Thus, in the $r_c \to \infty$ limit one goes from a 2D problem to a 1D one (as far as the value of the field on the brane is concerned), and one "looses" one dimension. This is analogous (but not strictly the same, as we will see) to what happens for DGP gravity and is another way to see how, in the full DGP model, one goes from a 5D theory to a 4D one. However, as we will see in the following, things are in fact much more involved when one considers the full DGP gravity model. In that case, indeed, if one study a Cauchy problem for linearized DGP, one does not find a recovery of the solution to the Cauchy problem associated with linearized 4D General Relativity when the brane space-time in Minkowski. This is due to the presence of the vDVZ discontinuity. However, such a recovery holds true when the brane world space-time is FLRW (with nonvanishing Ricci scalar). This will be discussed in the next section. We note further, that in the $r_c \to \infty$ limits, the limiting solution on the brane given by (23) provides then a Dirichlet boundary condition (in contrast with (18)) that can then be used to solve for the bulk solution. The solution so-obtained is matching the limiting one obtained from the exact expression (21).

3 Scalar Cosmological Perturbations in the DGP Model

3.1 General Formalism and Mukohyama's Master Variable

The most general scalar perturbations of the line element (13) read

$$g_{AB} = \begin{pmatrix} -n^2(1+2\bar{A}) & a^2\partial_i\bar{B} & n\bar{A}_y \\ a^2\partial_i\bar{B} & a^2[(1+2\bar{\mathcal{R}})\delta_{ij}+2\partial^2_{ij}\bar{E}] & a^2\partial_i\bar{B}_y \\ n\bar{A}_y & a^2\partial_i\bar{B}_y & 1+2\bar{A}_{yy} \end{pmatrix}, \tag{26}$$

out of which one can define four gauge-invariant quantities. For our discussion, it turns out convenient to use Mukohyama's Master variable formalism [41], that we now recall.

Mukohyama showed that all the linearized scalar Einstein's equations over a maximally symmetric background bulk[6] can be conveniently solved introducing a master variable Ω which obeys a PDE in the bulk, the master equation. The latter, when Ω has a nontrivial dependence in the comoving coordinates x^i, reads in the GN coordinate system (13)

$$\left(\frac{\dot{\Omega}}{na^3}\right)^{\cdot} - \frac{n\Delta\Omega}{a^5} - \left(\frac{n\Omega'}{a^3}\right)' = 0, \tag{27}$$

where $\Delta \equiv (\partial_i)^2$. In the rest of this work, we will implicitly consider all the perturbations as Fourier transformed with respect to the x^is, in order to do a mode by mode analysis. In particular (27) can be rewritten as

$$\mathcal{D}_\Delta \Omega = 0, \tag{28}$$

where $\mathcal{D}_\Delta$ is a second order hyperbolic differential operator acting on y and t dependent functions (in the GN system), and Δ is understood to be replaced by $-\vec{k}^2$, where $\vec{k}$ is the comoving momentum. The gauge invariant scalar perturbations can be expressed in term of linear combination (with background dependent coefficients) of Ω and its derivatives. Similarly, if one chooses a gauge where the perturbed brane metric takes the (*longitudinal*) form

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Psi)\delta_{ij}dx^i dx^j, \tag{29}$$

[6] In our case the background bulk space-time is just a slice of Minkowski space-time.

then Φ and Ψ as well as the perturbations of the matter energy momentum tensor $\delta\rho_{(M)}$, $\delta q_{(M)}$, $\delta P_{(M)}$ are expressed as linear combination (with background dependent coefficients) of Ω and its derivatives. This holds true if one assumes (as will be done in the rest of this work) that the matter anisotropic stress perturbation $\delta\pi_{(M)}$ vanishes. For example, Φ is given by (here and in the following the index (b) means that the corresponding expression is taken on the brane)

$$\Phi = \frac{1}{6a_{(b)}} \left\{ \frac{\Delta\Omega}{a^2} \mathcal{C}^{\Phi}_{\Delta(0,0)} + H\Omega^{\cdot}\mathcal{C}^{\Phi}_{(1,0)} + \Omega^{\cdot\cdot}\mathcal{C}^{\Phi}_{(2,0)} + H\Omega'\mathcal{C}^{\Phi}_{(0,1)} \right\}_{(b)}. \tag{30}$$

Ψ, $\delta\rho_{(M)}$, $\delta q_{(M)}$, and $\delta P_{(M)}$ are given by similar expressions, where the coefficients $\mathcal{C}$ (this holds also for the above equation) are depending only on the background, and have been computed in [11]. We note here incidentally that despite the assumed vanishing of the matter anisotropic stress $\delta\pi_{(M)}$, one finds that Φ and Ψ are not equal in general. In fact, in the gauge considered here, the difference between Φ and Ψ is expressed as

$$\Phi - \Psi = r_c^{-1}\tilde{\xi}, \tag{31}$$

$$= \zeta(t)\delta\pi_{(\mathcal{E})}, \tag{32}$$

where $\tilde{\xi}$ is the gauge invariant brane position, ζ is a background dependent coefficient that can be obtained from [20], and $\delta\pi_{(\mathcal{E})}$ is the anisotropic stress of the so-called Weyl fluid. This contrast with 4D general relativity, where the vanishing of the matter anisotropic stress (that we assumed here), implies the equality between Φ and Ψ. This nonvanishing of the difference between Φ and Ψ is intimately related to the vDVZ discontinuity, as will be made clearer in the following two subsections. This means in particular that one cannot consistently neglect the Weyl fluid contribution in the cosmological perturbation theory of DGP gravity without modifying the model itself. This Weyl fluid part is bothering, because it does not allow to solve for the evolution of the perturbations solving only equations along the brane. This has lead several authors to neglect this contribution in order to obtain a close system of equation on the brane (see e.g. [37, 38]). However, as should be clear from (31), this is not a consistent procedure, since the same “approximation” would result in fact in a disappearance of the vDVZ discontinuity on a flat background, already in the linear perturbation theory. This would mean that one has in fact modified the original model of DGP. Rather, one should keep the Weyl fluid contribution in order to compute cosmological perturbations [39, 40]. Not surprisingly, the two approaches do not agree on the physical predictions.

3.2 vDVZ Discontinuity on Minkowski Background

Let us first consider the case of a nonrelativistic source which only nonvanishing component of energy momentum tensor is the energy density perturbation $\delta\rho_{(M)}$. We further consider the space-time on the brane to be Minkowski. In this case (neglecting time derivatives), the relation between the gravitational potentials Φ and Ψ, and the master variable Ω reduces to

$$\Phi = \frac{4}{3}\varphi_{(b)}, \tag{33}$$

$$\Psi = \frac{2}{3}\varphi_{(b)}, \tag{34}$$

where we have defined φ by $4\varphi \equiv \Delta\Omega$. The master equation (27) together with the relation between $\delta\rho_{(M)}$ and Ω can be recast in the unique equation

$$\frac{1}{\kappa_{(5)}^2}(\Delta\varphi + \varphi'') + \frac{1}{\kappa_{(4)}^2}\delta(y)\Delta\varphi = \frac{1}{2}\delta(y)\delta\rho_{(M)}. \tag{35}$$

The above equation is the same as (10) and was solved in Ref. [7]. Considering a localized source, then $\varphi_{(b)}$, as given by (35), was found to vary for distances to the source much smaller than r_c as a usual four dimensional potential verifying (at small distances) the standard Poisson equation (with standard normalization)

$$\Delta\varphi_{(b)} = \frac{\kappa_{(4)}^2}{2}\delta\rho_{(M)}, \tag{36}$$

while at larger distances it was found to vary as a 5D potential (see [7]). One then recovers from the above discussion the two-folded manifestation of the vDVZ discontinuity: keeping r_c much larger that the distance to the source, one sees first from (33) and (36) that the gravitational potential on the brane Φ is renormalized by a factor $4/3$ with respect to the usual one. This has been discussed below (4). Secondly, one has $\Psi = \frac{1}{2}\Phi$, which matches analysis of the Schwarzschild solution in the DGP model [7, 23, 30, 34, 35, 42, 43] as well as the tensorial structure of the graviton propagator.

It is also instructive to discuss the case of time dependent perturbations over a Minkowski background[7] and contrast again the results with those of 4D general relativity. There, if we set $\delta\pi_{(M)}$ and the pressure perturbation, $\delta P_{(M)}$, to zero, one finds that Φ equals Ψ at all time and is given by a affine function of time. If one further demands that all matter source vanish at some initial time (that is to say that $\delta\rho_{(M)}$ and $\delta q_{(M)}$ vanish at initial time), then the initial values of Φ and $\dot{\Phi}$ have to vanish thanks to the initial constraints, meaning further that Φ vanish at all later time. As is well known, there are no scalar gravitational waves in 4D GR. This is no longer true in DGP gravity. Indeed, on a Minkowski background, the master equation (27) takes the form of a usual 5D d'Alembertian equation

$$\ddot{\Omega} - \Omega'' + \vec{k}^2\Omega = 0, \tag{37}$$

a solution of which can be taken to be simply the "zero mode"

$$\Omega = A\sin\left(\omega(t - t_0) + \omega_0\right), \tag{38}$$

with $\omega^2 = \vec{k}^2$. On can check that this solution verifies at all times the boundary condition for an empty brane and results in a nonzero Φ on the brane (obtained from (30)). On the other hand the initial values Φ_{t_0} and $\dot{\Phi}_{t_0}$ can be chosen arbitrarily. There are thus propagating scalar gravitational waves on the brane in the linearized DGP model. Those waves verify $\Psi = -\Phi$ and their very existence is at odds with 4D GR. This will also contrast with the results summarized in the next subsection.

3.3 General Cosmological Background: Differential Problem and Wellposedness

When dealing with a more general cosmological background, the problem we are interested in becomes the following: given specified initial conditions (that one has to provide both on

[7] Time dependent aspects of the vDVZ discontinuity of the Pauli–Fierz theory have been studied in Ref. [45].

the brane and in the bulk), say at a given cosmological time t_0, what are the perturbations as seen by a brane observer at a later cosmological time $t \geq t_0$? To answer this question for a given mode $\vec{k}$ requires solving the 2D PDE (28) with initial (Cauchy or also possibly characteristic) data provided along some initial curve in the 2D (y, t) plane, and, given the hyperbolic nature of the problem, some boundary condition for Ω along the brane.

In general there is not a clear way to give a boundary condition on the brane, since this requires solving the equations of motion for the perturbations of brane localized fields, which in turn are intricated with the metric perturbations. However in the simplest case of matter with vanishing anisotropic stress, and simple equation of state, one can obtain easily such a boundary condition [20, 44]. The latter takes the form

$$0 = \sum_{r=0}^{r=4} \beta_{(r,0)} \partial_t^r \Omega + \sum_{r=0}^{r=4} \beta_{(r,1)} \partial_t^r \Omega', \tag{39}$$

where the β are time dependent coefficients known from the background cosmological solution, and can be expressed as linear combinations of the coefficients $\mathcal{C}$. A boundary condition of identical form is found in the case where the only matter on the brane is a scalar field. In the following we will restrict ourselves to the here-above mentioned cases where a boundary condition of the form (39) holds.

Equations (27) and (39) are then all what is needed to solve for the evolution of DGP brane world cosmological perturbations once initial conditions are supplied in the bulk. One might worry about the nonstandard form of the boundary condition (39) which involves derivatives of the master variable along the brane (such a boundary condition has been called *nonlocal* [44]). However, one can recast the differential problem defined by (27) and (39) in a standard form [21]. For more details on this rewriting, we refer to the original reference [21]. Standard theorems on linear differential systems then insure that the Cauchy problem, associated with the system we are looking at, is well posed.

3.4 Disappearance of the vDVZ Discontinuity on FLRW Background with Nonvanishing Scalar Curvature

We are now in a position to discuss the vDVZ discontinuity on a FLRW background. We want to study the behavior of cosmological perturbations of the DGP model where r_c is send to infinity with respect to a background cosmological length scale, that we will take to be the Hubble radius H^{-1}. In this limit: (i) we want to see whether initial data for the brane quantities Φ, Ψ, $\delta\rho_{(M)}$, $\delta q_{(M)}$, $\delta P_{(M)}$, can be specified arbitrarily close to the initial data for the corresponding quantities of usual 4D cosmological perturbations (or to say it in an other way, if the initial constraints of 4D GR can be fulfilled), and, (ii) we want to know if the evolution of those initial data differ, or not, from the one given by the theory of standard 4D cosmological perturbation. These two questions are the ones an observer living on the brane would like to answer in order to compare in a cosmological context the predictions of the linearized DGP model to the ones of linearized 4D GR.

Thus, the limit we will subsequently consider is $Hr_c \to \infty$, that is to say $\Upsilon \to 0$, keeping H, $\kappa_{(4)}^2$, as well as the other 4D background quantities $\rho_{(M)}$ and $P_{(M)}$, finite. This because we want to compare cosmological perturbations of the DGP model with the ones of standard GR in the same cosmological background. This limit will be denoted simply by $\Upsilon \to 0$.

We will not give here a full detailed discussion of this limit, but only underline the most important aspects of it. For more details we refer the reader to the original reference [11]. Let us first discuss the case of the background space-time. As we already explained below (16), in the $\Upsilon \to 0$ limit, the brane Friedmann (14–15) go to the standard Friedmann equations

$$\dot{\rho}_{(M)} = -3H(P_{(M)} + \rho_{(M)}), \tag{40}$$

$$3H^2 = \kappa_{(4)}^2 \rho_{(M)}, \tag{41}$$

obtained simply by making $\Upsilon = 0$ in the system (14–15). As noted in [8], there is no discontinuity in the background cosmological solution. The solution to (40–41) define a trajectory in the (y, t) plane, to which the brane (denoted as $\mathcal{C}_{(b)}$) with no zero Υ can be made arbitrarily close by letting Υ go to zero.

Let us now turn to the behavior of the perturbations themselves. The associated differential problem is entirely specified by the master equation (27), the boundary condition (39), and a domain $\mathcal{D}$ bounded by the brane $C_{(b)}$ and the initial curve $C_{(I)}$ on which we give initial data. Among those different elements, we have just discussed the behavior of $C_{(b)}$ in the $\Upsilon \to 0$ limit. As far as $C_{(I)}$ is concerned, we pick a fixed noncharacteristic (a similar discussion with identical conclusions can easily be carried out when the initial curve is characteristic) initial curve in the bulk, independently of Υ (which we can always do). Note that, when Υ does not vanish, the domain $\mathcal{D}$ can be made arbitrarily close to the limiting domain bounded by the chosen initial curve and the limiting curve $C_{(b)}^{\Upsilon\to 0}$. The master equation does not depend on Υ, so the only Υ-depending pieces that we have not discussed are the choice of initial data on the initial curve $C_{(I)}$ and the boundary condition (39). Let us now turn to these last two issues.

We first note one crucial result: assuming that the background Ricci scalar of the induced metric on the brane, $R_{(b)}^{(B)}$, does not vanish on the time interval $[t_0, t_{\max}]$, one can show that all of the coefficients $\mathcal{C}$ have a well behaved and bounded limit. A first consequence of this is that the boundary condition (39) has a well defined limit when $\Upsilon \to 0$.

We turn then to the choice of initial conditions. We introduce first a new coordinate system $\tau(t, y)$ and $z(t, y)$ such that the curve $C_{(I)}$ is defined by a constant z, and that the $\tau =$ constant curves are normal to $C_{(I)}$. For a fixed Υ, we can choose arbitrarily on $C_{(I)}$ the values of Ω and of its normal derivative $\partial_z\Omega$ w.r.t. $C_{(I)}$. The only constraint these values have to obey comes from requiring their compatibility with the boundary condition (39) in O, which depends on Υ. In addition, having in mind a comparison with cosmological perturbations of 4D GR, we wish to fix at the initial time t_0 on the brane (defined to be the event O), the values Φ_{t_0} and $\dot{\Phi}_{t_0}$ (of Φ and $\dot{\Phi}$ respectively). We thus impose three constraints on the initial conditions at O, which in turn impose constraints on the value of Ω and of its derivatives at O, through (39), (30), and the derivative of (30) with respect to t. One can then show that these constraints can be simultaneously imposed at O, simply by choosing the values of Ω and $\partial_z\Omega$ on $C_{(I)}$. We call $\mathcal{S}$ a set of initial conditions (for $\Upsilon \in$ some interval $]0, \Upsilon_{\max}]$)

$$\mathcal{S} = \{(\Omega_{(I)}^{\Upsilon}, \partial_z\Omega_{(I)}^{\Upsilon}), \Upsilon \in]0, \Upsilon_{\max}]\}, \tag{42}$$

that has the property that its elements $(\Omega_{(I)}^{\Upsilon}, \partial_z\Omega_{(I)}^{\Upsilon})$ fulfill the boundary condition (39) in O, give fixed values Φ_{t_0} and $\dot{\Phi}_{t_0}$ for Φ and $\dot{\Phi}$ at O, and go to a well defined limiting initial condition when Υ is send to zero. We call $\Omega_{\mathcal{S}}^{\Upsilon}$ the solution over $\mathcal{D}$ to the master equation (27) with the initial data $(\Omega_{(I)}^{\Upsilon}, \partial_z\Omega_{(I)}^{\Upsilon})$ chosen in a given set $\mathcal{S}$ and obeying the boundary condition (39). The here-above noticed fact that the boundary condition (39) has a well behaved limit when $\Upsilon \to 0$, as well as the fact that the differential problem we are looking at is well posed [21], shows that the solution $\Omega_{\mathcal{S}}^{\Upsilon}$ has a well defined limit, $\Omega_{\mathcal{S}}^{\Upsilon\to 0}$, when $\Upsilon \to 0$. Once one knows this limit, one can compute from (30), and those similar for Ψ, $\delta\rho_{(M)}$, $\delta q_{(M)}$, $\delta P_{(M)}$, the limiting value (as $\Upsilon \to 0$) at all time $t \in [t_0, t_{\max}]$ of the brane expressions Φ, Ψ, $\delta\rho_{(M)}$, $\delta q_{(M)}$, $\delta P_{(M)}$. The question of the vDVZ discontinuity then translates into asking how these limiting values, noted $\Phi^{\Upsilon\to 0}$, $\Psi^{\Upsilon\to 0}$, $\delta\rho_{(M)}^{\Upsilon\to 0}$, $\delta q_{(M)}^{\Upsilon\to 0}$, $\delta P_{(M)}^{\Upsilon\to 0}$, compare with the corresponding quantities of standard 4D GR, than is to say the solutions of the linearized 4D Einstein equations with the same initial conditions Φ_{t_0} and $\dot{\Phi}_{t_0}$.

To answer this question, we first define new quantities, that we will denote respectively Φ^0, Ψ^0, $\delta\rho^0_{(M)}$, $\delta q^0_{(M)}$, $\delta P^0_{(M)}$, by taking the limiting values in $\Upsilon \to 0$ of the right hand side of (30) and those similar giving Ψ, $\delta\rho_{(M)}$, $\delta q_{(M)}$, $\delta P_{(M)}$ as function of Ω and its derivatives. E.g. Φ^0 is defined by

$$\Phi^0 = \frac{1}{6a_{(b)}}\left\{\frac{\Delta\Omega}{a^2}\mathcal{C}^{\Phi}_{\Delta(0,0)} + H\Omega^{\cdot}\mathcal{C}^{\Phi}_{(1,0)} + \Omega^{\cdot\cdot}\mathcal{C}^{\Phi}_{(2,0)} + H\Omega'\mathcal{C}^{\Phi}_{(0,1)}\right\}_{(b),\Upsilon=0}, \tag{43}$$

where the index $_{\Upsilon=0}$ means that all the coefficients $\mathcal{C}$ are taken in $\Upsilon = 0$, but also that the quantities depending on the background brane trajectory (and thus also on Υ, as we saw above) are given by the limiting $C_{(b)}^{\Upsilon=0}$ trajectory. This means e.g. that the H entering into (43) is obtained by definition by solving the standard Friedmann (40–41). Note further that the function Ω entering into the above definition of Φ^0, Ψ^0, $\delta\rho^0_{(M)}$, $\delta q^0_{(M)}$, and $\delta P^0_{(M)}$ is so far left arbitrary. Now comes the crucial fact: one can verify after some straightforward, but tedious, algebra that the expressions Φ^0, Ψ^0, $\delta\rho^0_{(M)}$, $\delta q^0_{(M)}$, $\delta P^0_{(M)}$ verify identically the standard 4D perturbed Einstein's equations with $\delta\pi_{(M)} = 0$, for arbitrary Ω and Ω' (in particular Φ^0 equals Ψ^0). This answers the question, since it shows that for any set of initial condition $\mathcal{S}$ (and thus any limiting solution $\Omega_{\mathcal{S}}^{\Upsilon\to 0}$) the limiting expressions $\Phi^{\Upsilon\to 0}$, $\Psi^{\Upsilon\to 0}$, $\delta\rho_{(M)}^{\Upsilon\to 0}$, $\delta q_{(M)}^{\Upsilon\to 0}$, $\delta P_{(M)}^{\Upsilon\to 0}$ are exactly equal to the corresponding solutions of cosmological perturbations computed from 4D GR, with the same initial conditions. Thus the latter can not be distinguished from the former by a brane observer. In this sense, it shows that there is no vDVZ discontinuity on a FLRW space-time with nonvanishing Ricci scalar curvature (with the restriction that our proof is only valid for matter with vanishing anisotropic stress and equation of state leading to a boundary condition of the form (39)).

An other way to look at this result is to say that in the $\Upsilon \to 0$ limit, the knowledge at O of Φ and $\dot{\Phi}$ and of the brane matter equation of state are enough to compute on $C_{(b)}$ the values of Φ, Ψ, $\delta\rho_{(M)}$, $\delta q_{(M)}$, $\delta P_{(M)}$ at all time $t \in [t_0, t_{\max}]$. So that the brane decouples from the bulk in this limit.

4 Discussion and Conclusion

In this paper, we mostly reported on results obtained in [11] on linear cosmological perturbations of the Dvali–Gabadadze–Porrati model and the vDVZ discontinuity. In the case of a brane background space-time with a nonvanishing Ricci scalar, we showed that if one fixes at some initial time along the brane world-volume, the initial values of the gravitational potential Φ and its time derivative $\dot{\Phi}$, as can be done in 4D general relativity, and if one further insists that the initial data to be provided in the bulk have a well-behaved (continuous and bounded) limit when we let the radius r_c of transition between the 4D and 5D regime go to infinity with respect to the Hubble radius of the background space-time, then the linearized theory on the brane does have a limit, and this limit is given by the theory of cosmological perturbations derived from standard 4D GR. This holds for the simplest case of matter living on the brane, namely a perfect fluid with adiabatic perturbation, or a scalar field. This result can be rephrased in saying that the boundary condition on the brane, derived from the matter equation of state, and that is usually "nonlocal" in time [44], degenerates, in the limit considered, into a standard Dirichlet boundary condition that is obtained solving the linearized 4D Einstein's equations. So, in fact, in this limit, the brane decouples from the bulk. It contrasts with the case of a Minkowski background, and can be interpreted as showing, in a "Hamiltonian" sense, that the vDVZ discontinuity does disappear on a FLRW background in a similar way as it does on a maximally symmetric background with a non-vanishing curvature. What we mean here by that, is that an observer living on the brane, and considering only scalar perturbations, thus, say, fixing some initial data as he would do in standard 4D cosmology, will not be able to distinguish the time evolution of these data in the DGP model with large enough $r_c H$, from what he would have observed in standard cosmology with the same initial data. The same result is found to hold in the case of a brane background space-time with a vanishing Ricci scalar, with however the important difference that the initial data in the bulk are to be such that their limiting values vanish (see [11]). This contrasts with the previous case where the only constraint those bulk initial data have to fulfill are those implied by the choice of initial data on the brane. Thus, in the case of a brane with a vanishing Ricci scalar, the limiting perturbations are solely supported by the brane motion in a bulk empty from metric perturbations. Moreover, as we argued in [11], the vDVZ discontinuity somehow reappears on a cosmological background where the Ricci scalar does not vanish but on a measure zero subset. In this case, if one only considers the time evolution of perturbations given by the linearized theory and follows the procedure outlined above for fixing initial conditions, one finds divergences in observable quantities at the points where the Ricci scalar vanishes when one lets $r_c H$ go to infinity. These divergences result in a breaking down of the linearized theory, in a similar way to what is happening around a static source on a Minkowski background [23, 30, 33–35], which is intimately related to the vDVZ discontinuity. One might be concerned that our results seem to depend on the assumption the bulk initial data have a continuous and bounded limit. In fact, the only necessary condition is that those bulk initial data are such that the limiting value (as $r_c H$ goes to infinity) on the brane of the master variable and its derivatives are bounded. When it is not the case, we argued in [11] that the linear perturbation theory is no longer valid. It would be interesting to generalize the approach adopted here to the full nonlinear theory in an effort to prove the disappearance of the vDVZ discontinuity in an initial value formulation.

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